14.3 Partial Derivatives — Meaning and Notation

Recall: A partial derivative measures slope when one input changes and the other is held fixed.

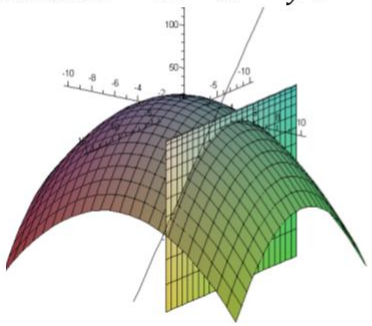
$$f_x(x,y) = \frac{\partial f}{\partial x} \qquad f_y(x,y) = \frac{\partial f}{\partial y}$$

f_x is the slope in the x-direction (hold y constant).

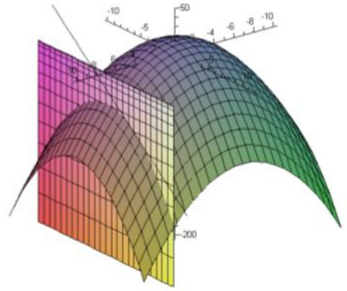
f_y is the slope in the y-direction (hold x constant).

Visualizing slopes on a surface

The plane y = 4 intersecting the surface $z = 15 - x^2 - y^2$.



The plane x = 7 intersecting the surface $z = 15 - x^2 - y^2$.



https://www.desmos.com/3d/esqi9npfuf

Entry Task

Let $f(x,y) = 15 - x^2 - y^2$

and find the $f_x(7,4)$ and $f_y(7,4)$.

Second Partial Derivatives

2nd deriv. describe how the 1st deriv. change:

Concavity:

$$f_{xx} = \frac{\partial}{\partial x}(f_x) \quad , \quad f_{yy} = \frac{\partial}{\partial y}(f_y)$$

Example: Find all first and second partials of

$$f(x,y) = x^2 - y^2$$

Clairaut's Theorem: If the mixed second partials are continuous near the point, then $f_{xy} = f_{yx}$ at that point.

Mixed Partials

$$f_{xy} = \frac{\partial}{\partial y}(f_x) \quad , \quad f_{yx} = \frac{\partial}{\partial x}(f_y)$$

Example: Find all four second partial derivatives for

$$f(x,y) = x^4 + 3x^2y^3 + y^5$$

https://www.desmos.com/3d/kjk9bvrczo

https://www.desmos.com/3d/abxxggcw1k

Computing Partial Derivatives · *HW Part 1 Q3–8*

Use the same differentiation rules as Calculus I. The other inputs are constants.

Example 1: Quotient Rule *HW Q4*

$$w(u,v) = \frac{e^{v}}{u + v^{6}}$$

Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$

Example 2: Three Variables *HW Q8*

$$u(x,y,z) = 4x^{y/z}$$

Find u_x, u_y, and u_z.

Calculus - Summary of Derivatives

A variable may be treated as:

1. A constant
2. An independent variable (input)
3. A dependent variable (output)

Example 1: $y = x^2$, find $\frac{dy}{dx}$.

Example 2: An object location is given by $(x(t), y(t))$ and always moves along the curve $y = x^2$, find $\frac{dy}{dt}$.

Example 3: $x^2 + xy^2 = 1$. Find $\frac{dy}{dx}$

Example 4: $z = x^2 + y^3 - 5xy^4$

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Example 5: $x^2 + xy^2 - z^2 = 1$

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

Interpreting Partial Derivatives *HW(2) Q1–3, 8–9*

At a given point on a surface, what do the signs of f_x and f_Y tell us?

Positive:

increasing height in the positive coordinate direction.

Negative:

decreasing height in that direction.

Second partials:

concavity along coordinate directions;

Mixed partials:

how one slope changes as the other coordinate varies.

App: Temperature on a Plate · *HW(2) Q8*

$T(x, y) = 65/(3 + x^2 + y^2)$ degrees Celsius

At (2,3), find the rate of temperature change in the x-direction and in the y-direction. Include units.